Sanlam Wealthsmiths™

presenting foreign asset returns in rand terms

Investments

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Estimating future foreign asset returns in Rands via convolutions.

Introduction:

Assessing investment opportunities for foreign asset classes is often based on their performance, measured in local currency. This is usually done by multiplying the foreign returns with the relevant exchange rate, and then using these domestic returns as the basis for making asset allocation decisions.

This paper shows how using the traditional method of multiplying foreign asset performance with the relevant currency exchange as the basis for foreign asset allocation can be misleading.

There are additional considerations that investors may miss when making decisions about how much and where to invest offshore when the traditional method is used. In contrast to this, using the convolutions method, for determining foreign asset performance in local currency, allows investors to consider three distinct factors:

- The varying returns of the foreign asset.
- The currency fluctuations when converting to domestic currency.
- The dependency structure (e.g. correlation coefficient) between the foreign asset and the exchange rate in question.

Ultimately, this report aims to show that investors should make use of a convolution method to incorporate these three elements correctly when making robust decisions regarding the assessment of foreign opportunities.

Background[1]:

Recent changes to Regulation 28 have had definite impacts on the investment landscape, with investors (both institutional and retail) now able to increase their offshore asset allocations from 25% to 30%. This regulatory change opens up questions about exactly how much investors should allocate offshore and where they should invest their money (for example, what assets should be chosen, and how can we compare their domestic returns?). Our aim is to provide a framework where investors can evaluate these questions while considering exchange rate views in their portfolio construction process. We mainly consider the perspective of an investor who intends to invest offshore, but receives returns in Rands.

It is natural that when investors assess their investment returns, the distribution of the actual returns should be used, i.e. foreign asset returns in local currency terms. The distribution of these returns is generally used in the formulation of the strategic asset allocation, predominantly created through a mean-variance optimisation routine. This allows investors to determine the appropriate proportion of capital that they can allocate to foreign assets. Furthermore, investors will often evaluate various risk metrics such as expected variance, from the distribution of actual returns.

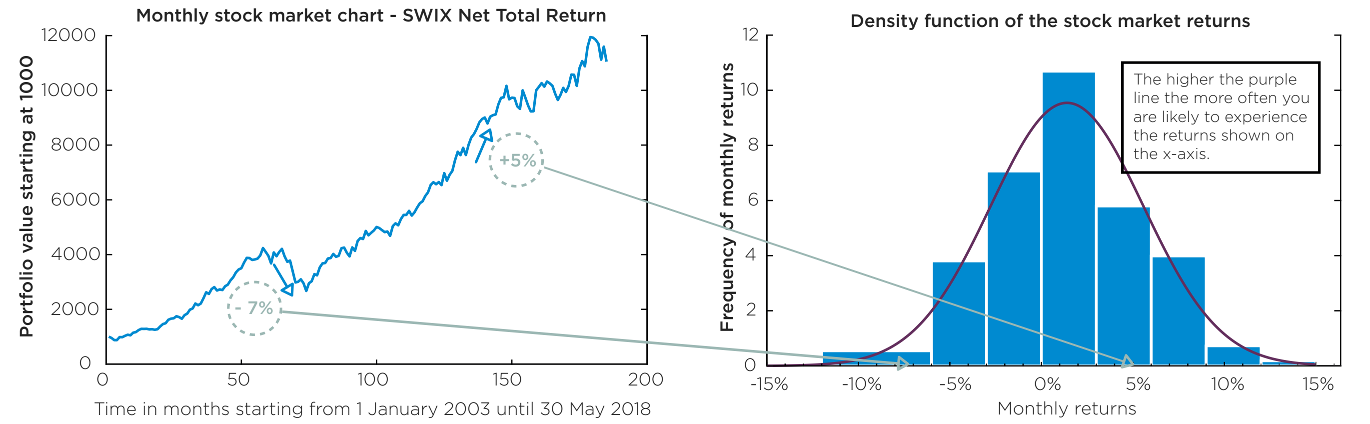
Essentially this report will show that it is preferable to **forecast the foreign asset's returns and the currency's returns separately**, combining them thereafter for an appropriate foreign asset forecast. This approach can be extended to any asset and currency pair, and should be done instead of specifying return and/or volatility forecasts for a foreign asset in Rand terms directly.

[1]See appendix for detailed mathematical explanation.

Basic Investment Return Models

Investors experience their returns as an increase/decrease in the initial wealth that they invested. When investors evaluate their investments, they are likely to see the following type of chart (the chart has a monthly frequency, but the time frame could also be daily, weekly or quarterly).

FIGURE 1: Snapshot of monthly market data, and distribution chart showing the frequency of changes to the investment.



Source: Sanlam Investments, July 2018

Figure 1. (left) shows how the portfolio of an investor who invested R1 000 on 1 January 2003 into the SWIX, has appreciated over time. At each point in time we can take the returns experienced by this investor every month and plot a histogram of those returns (right). This represents the density function of returns, a method of associating what returns an investor can experience with how often they are likely to experience this.

More formally, we can say that the variable[2] X, representative of the monthly returns of an asset, follows a specified distribution (a statistical law providing information about *X*).

Traditionally determining local returns distributions for foreign assets

Investors traditionally estimated the **foreign asset in local currency "Z"** returns distribution (rage and frequency of returns changes) as: (one plus the **asset returns "X"** distribution) multiplied with (one plus the **currency pair returns "Y"** distribution)

$$1 + z = (1 + x) * (1 + y) \qquad (1.1)$$

Practical example

As a practical example, let's assume that our foreign asset is an index fund tracking the MSCI ACWI (an index which captures large and mid-cap representation across 23 developed markets and 24 emerging markets countries) in dollar terms. Also assume that the exchange rate in question is given by the USDZAR currency pair. A summary of the returns distribution (using the last 269 months' worth of data from January 1996 until May 2018) is shown below for the presumed traditional relationship between the currency pair and asset returns highlighted previously (Formula 1.1).

Table 1. Summary of the distribution results for the asset returns, currency pair returns and the resulting asset in local currency returns, using the traditional method.

The **geometric mean returns** specify how much the asset has generated for the investor on a yearly basis over the life of the investment, while the standard deviation specifies in broad terms how volatile these returns have been. The **arithmetic mean returns** and **standard deviation** of returns together yield an **efficient frontier** – a useful aid in determine an investor's asset allocation. The plot of the distributions for each of the assets is given in the following graph[3].

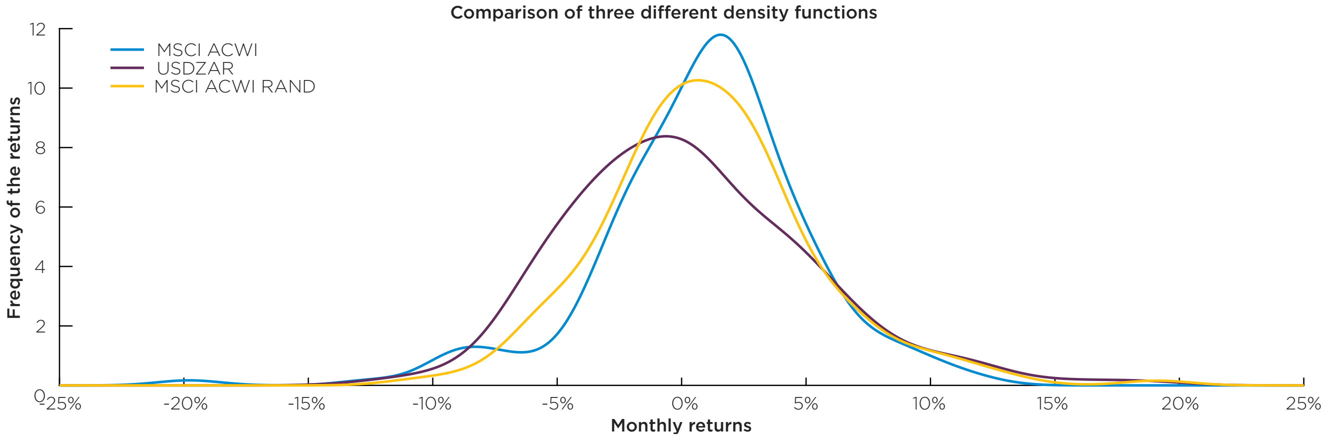
[2]A random variable is simply the name we assign to some quantity to denote the fact that this quantity can have random values. The density function then gives us information about this randomness, i.e. returns around 1% a month are more likely than 10% or -10%.

[3]Our estimation of each of the density functions from the samples was undertaken by using a Kernel density estimation technique (estimating what statisticians call the empirical density function). This means that we do not strictly use a statistical model to describe the monthly returns, but we rather let the data speak for itself.

TABLE 1:

	Arithmetic Mean Return	Geometric Mean Return	Standard Deviation of returns	Skewness of returns	Kurtosis of returns
MSCI ACWI ($)	8.06%	7.11%	15.21%	-0.76	4.91
USDZAR Currency	6.82%	5.72%	15.97%	0.63	4.40
MSCI ACWI (RAND)	13.83%	13.24%	16.49%	0.41	4.43

FIGURE 2: Comparing the distribution of returns for the foreign asset (MSCI ASWI), dollar to rand exchange returns (USD ZAR) and foreign asset in local currency returns (MSCI ASWI RAND).



Source: Sanlam Investments, July 2018

While we support the traditional method for calculating foreign asset returns in local currency up to this point, the **problem is now that we have two sets of distributions** (asset returns distribution and currency pair distribution). Combining this double data set, made up of historical numbers, makes it difficult to apply forward-looking allocations as both the returns for the asset and returns for the currency pair are subject to change differently in future. And, when looking to allocate to a single asset, for example foreign equity, it is essential to be able to apply forward-looking projections to the data in order to estimate possible changes to the returns distribution of that asset. It is important to note that **for forward-looking estimations, multiplying values from each of the two distributions will not give the correct final distribution for the foreign asset in local currency** - as the traditional method of multiplication assumes that **the assets behave independently**[4], which is unlikely in practice.

Simply put, the aggregation of data via simple multiplication loses information and therefore could hide various risks or changes for each individual set of returns.

In essence, we have two sets of *N* datapoints, which become one set of *N* data points after applying (1.1). Using these *N* datapoints to then model a joint interaction without reference to the original data, hides risks and masks important interaction effects between the random variables. This is demonstrated in the next section where we formally introduce convolutions.

Using the convolution method to estimate future returns

Introduction to Convolutions

In order to introduce convolutions as a method of combining data for foreign asset returns with currency exchange data, we will first explore a theoretical example of combining two sets of random data via convolutions.

Assume that we have two random variables (X and Y) that are normally distributed with means of 4% and 6%, and standard deviations of 4% and 7%. Additionally, assume that the normal variables are negatively correlated with a correlation coefficient of -0.6.

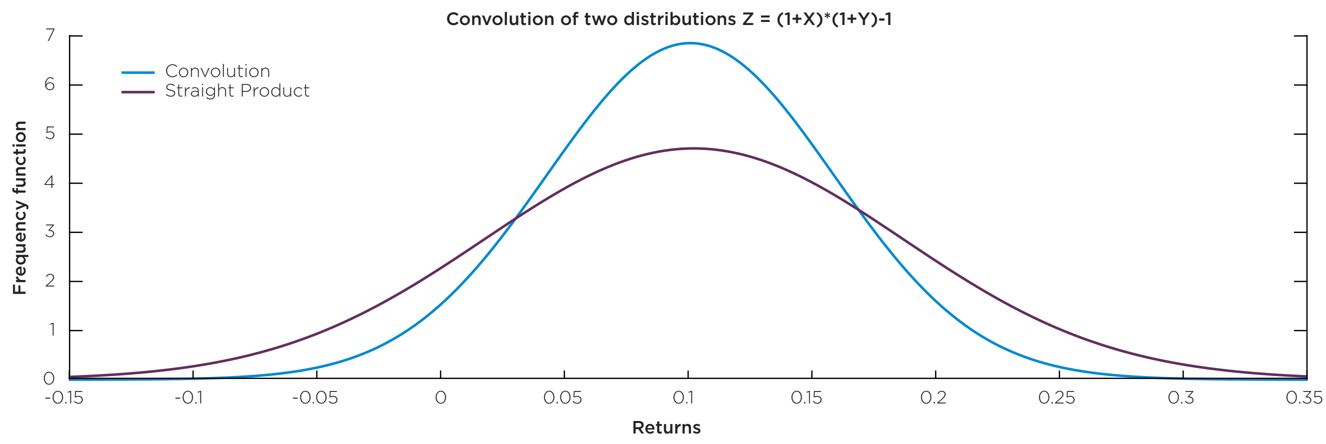
[4]See Table I in appendix.

In order to gain a more accurate estimation of combining the two variables, we perform a statistical procedure called a convolution of the two random variables. It is a mathematical procedure that allows us to correctly "mix" each of the individual data distributions while taking into account the dependency structure between these distributions. A convolution of the two theoretical data sets shown in Figure 2 yields the distribution curve in Figure 3 (see appendix for construction):

For a better understanding of how this graph is created, we created the following plot with simulated data points for each of the distributions:

If instead of performing a convolution, we made use of the traditional estimation formula, a very different result is observed. We drew 10 000 random variables from each of the distribution data sets individually and multiplied them together, as per the traditional formula, yielding a different distribution (purple line) than when we performed the more accurate convolution (blue line):

FIGURE 3: Comparison of the distribution curves yielded from the traditional and convulsion methods.



The reason for the discrepancy observed in Figure 3 is that the traditional method does not consider the joint behaviour of the assets (red line). As each asset has a unique distribution, we can increase the accuracy of the traditional method by additionally specifying the joint behaviour (distribution) of two or more assets to create the joint probability density function (pdf).

If we correctly specify the joint pdf, then **generating random values from this joint pdf** and **multiplying the results** will give a more accurate distribution, **similar to performing a convolution**. This leads to an important conclusion when working with the distributions of a foreign asset and its relevant currency pair[5]:

> *Specifying individual forecasts without taking into consideration how those forecasts will co-move with the other data set, will not give an accurate distribution. It is therefore important for institutional investors to consider that the modelling of individual foreign assets in rand terms cannot be separated from the modelling of how those assets behave jointly with a given currency pair.*

The following section includes theoretical results applied to our MSCI ACWI and USDZAR example. Thereafter we demonstrate how the convolution method above can be used to infer portfolio allocation decisions when making comparative assumptions about the assets.

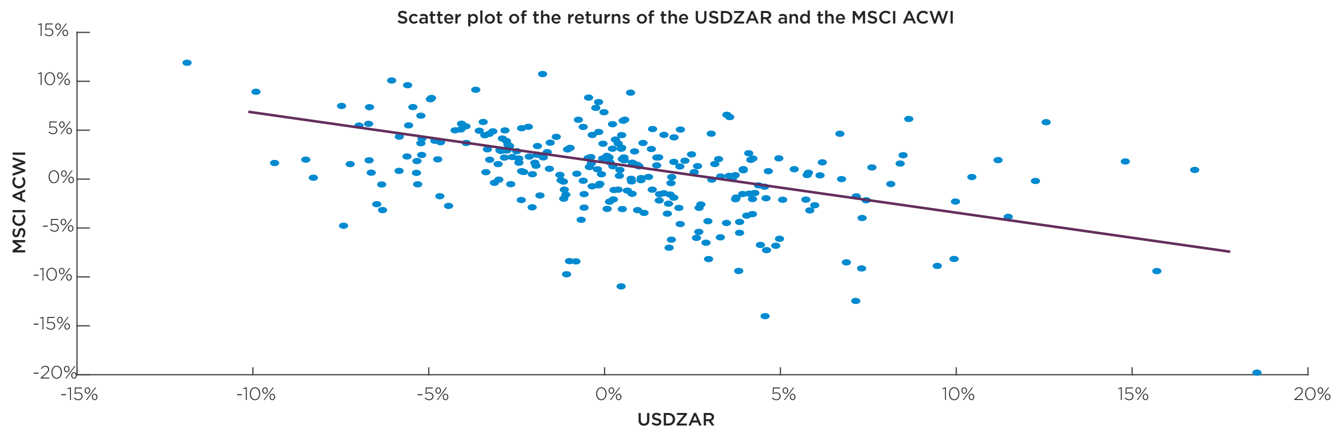
Theoretical Results:

Returning to our example of a foreign asset being the MSCI, to obtain a feel for how the MSCI ACWI and the USDZAR co-move, we compute the possible relationship between the two data sets, in order to determine their dependency . Figure 4 shows a considerable dependency between the returns, with a significant correlation coefficient of -0.4566.

[5]See appendix for the mathematical process.

[6]For demonstration purposes, we assume that both the USDZAR and the MSCI ACWI follow normal distributions and that we can model the dependency between them using a correlation coefficient.

FIGURE 4: Determining dependency between the foreign asset returns and currency exchange returns.



Now that we are aware of the dependency between the foreign asset returns and the exchange rate returns, we need to adjust the combined returns distribution (determined with historical data) for our expected future returns. For this example, we estimate that the MSCI ACWI will give lower returns in the coming years with a higher volatility, and we assume that the Rand/Dollar exchange rate will stabilise slightly[7].

Tables 2a and 2b. Summary of the historic returns distribution (Table 2a) and estimated future returns distribution (Table 2b) for the foreign asset and currency exchange.

TABLE 2A:

Historic Views	MSCI ACWI (DOLLAR)	USDZAR
Arithmetic Mean (Monthly)	0.67% [8.06%]	0.57% [6.82%]
Standard Deviation (Monthly)	4.39% [15.21%]	4.61% [15.97%]
Dependency (Correlation)	-0.45	-0.45

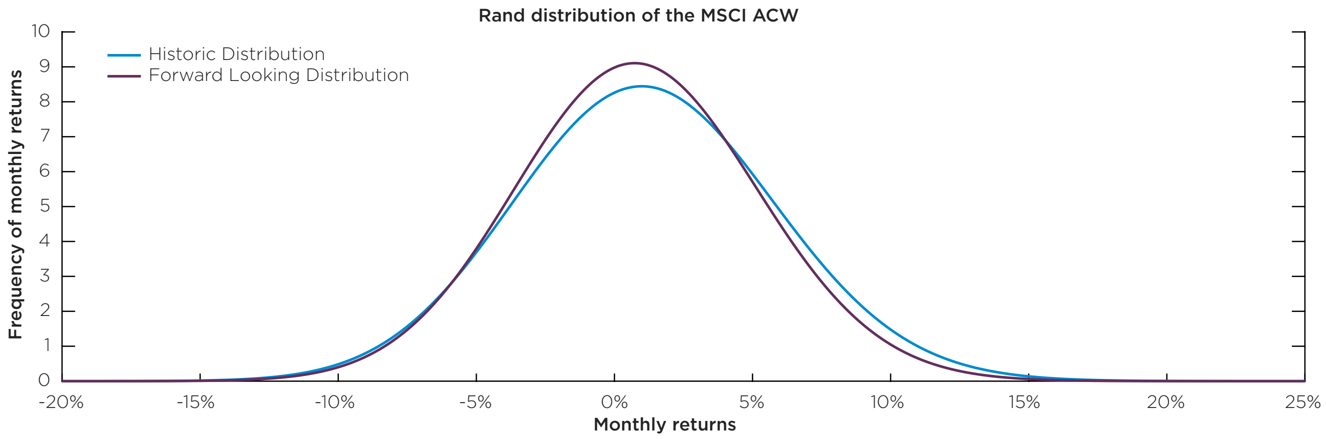
TABLE 2B:

Example Forward Views	MSCI ACWI (DOLLAR)	USDZAR
Arithmetic Mean (Monthly)	0.55% [6.60%]	0.40% [4.80%]
Standard Deviation (Monthly)	5.00% [17.32%]	4.00% [13.86%]
Dependency (Correlation)	-0.55	-0.55

Using the convolution method, we created a future view of the estimated returns distribution for the foreign asset in local currency. The difference between the estimated future returns and historic returns are shown in Figure 5.

[7]This estimation is based on current market trends and is not a certainty of future changes.

FIGURE 5: Comparison of the historic and projected future returns distribution of the foreign asset in local currency, using the convolution method.



From Figure 5 we see the joint effect on the Rand distribution of the MSCI ACWI returns. Effectively, the attractiveness of investing offshore based on estimated forward views has diminished slightly. Returns are more concentrated (higher peak) around the -5% to 5% mark, and the upward potential of returns in excess of 5% has been reduced.

Practical applications of the convolutions method:

These results allow us to appreciate the complex dynamics of working with a foreign asset and exchange rates, and how the effects of fluctuations in both sets of returns would impact out future view on asset allocation.

To stress the importance of using the convolution method to more accurately estimate future returns and a resulting choice of asset allocation, we will consider the effects of various exchange rate views on the returns of a foreign asset. We compare the returns distribution using the convolution method and the more traditional estimation for Rand weakening, stability and strengthening estimates.

TABLE 3:
The monthly and annualised historic figures for the MSCI ACWI (Dollar) and the USDZAR for the period February 2003 to April 2018.

Historic Views	MSCI ACWI (DOLLAR)	USDZAR
Arithmetic Mean	0.86% [10.34%]	0.32% [3.86%]
Standard Deviation	4.27% [14.80%]	4.87% [16.86%]
Dependency (Correlation)	-0.58	-0.58

This data shows that historically the appropriate choice of the correlation coefficient depended on the exchange rate environment as well as the length of time of the history of the data (compare to above example with data from January 1996 until May 2018). We find that the negative correlation does not vary significantly (i.e. between -0.6 and -0.5) when the Rand weakens or strengthens. However, there appears to be little correlation when the Rand is stable. Please note that this does not imply that the Rand and the MSCI ACWI behave independently when the Rand is stable.

The second step is to then split our exchange rate assumptions into three parts: Rand strengthening, weakening and stability. We then estimate our resulting forward views for each possibility. The tables below compare the estimated future returns of the MSCI ACWI given in Rand terms for each relative view of the Rand strength, using a) the traditional but less accurate method[8], and b) the more accurate convolution method.

[8]See Table 4 as an example of how the Future MSCI ACWI (R) – Historic Adjusted is calculated for the Rand stability view: Reduce the historic mean of 12.79% mean by ((0.86%-0.75%) + (0.32%-0.20%))*12 = 2.76%. This gives 10.03% as indicated in the table. Tables 5 - 7 follow similarly. Standard Deviation is estimated when making adjustments without using the convolution method as there is no efficient way to blend the two volatilities. This is likely to be kept the same as the historical standard deviation. Additionally, if one has any beliefs about the correlation coefficient between the exchange rate and the foreign asset then it is impossible to incorporate this information unless using the convolution method.

TABLE 4:

Estimated changes to possible rand views.

Asset	MSCI ACWI (DOLLAR)	USDZAR		
Example Forward Views	Historic	Rand Weakening	Rand	0.32% [3.86%]
Stability	Rand Strengthening	4.27% [14.80%]	4.27% [14.80%]	4.87% [16.86%]
Arithmetic Mean (Monthly)	0.75% [9.00%]	0.60% [5.00%]	0.20% [2.40%]	-0.20% [2.40%]
Standard Deviation (Monthly)	4.90% [16.97%]	5.50% [19.05%]	3.50% [12.12%]	3.00% [10.39%]
Correlation with MSCI World	-	-0.75	-0.2	-0.50

The tables below now show the impact of these views on the returns distribution of the MSCI ACWI in Rand terms. For each view we give the Historic MSCI numbers and thereafter we give the future numbers by adjusting our assumptions using the convolution method and then the traditional method to allow for a comparison of the methods.

TABLE 5:

Comparing the Rand weakening view for foreign asset returns in local currency using the convolutions method and more traditional method.

Historic Views	Historic MSCI ACWI (R)	Future MSCI ACWI (R) - Convolution	Future MSCI ACWI (R) – Historic Adjusted
Arithmetic Mean	12.79%	13.80%	14.83%
Standard Deviation	14.83%	13.03%	14.83%

TABLE 6:

Comparing the Rand stability view for foreign asset returns in local currency using the convolutions method and more traditional method.

	Historic MSCI ACWI (R)	Future MSCI ACWI (R) - Convolution	Future MSCI ACWI (R) – Historic Adjusted
Arithmetic Mean	12.79%	11.04%	10.03%
Standard Deviation	14.83%	18.84%	14.83%

TABLE 7:

Comparing the Rand strengthening view for foreign asset returns in local currency using the convolutions method and more traditional method.

	Historic MSCI ACWI (R)	Future MSCI ACWI (R) - Convolution	Future MSCI ACWI (R) – Historic Adjusted
Arithmetic Mean	12.79%	5.76%	5.23%
Standard Deviation	14.83%	14.83 %	14.83%

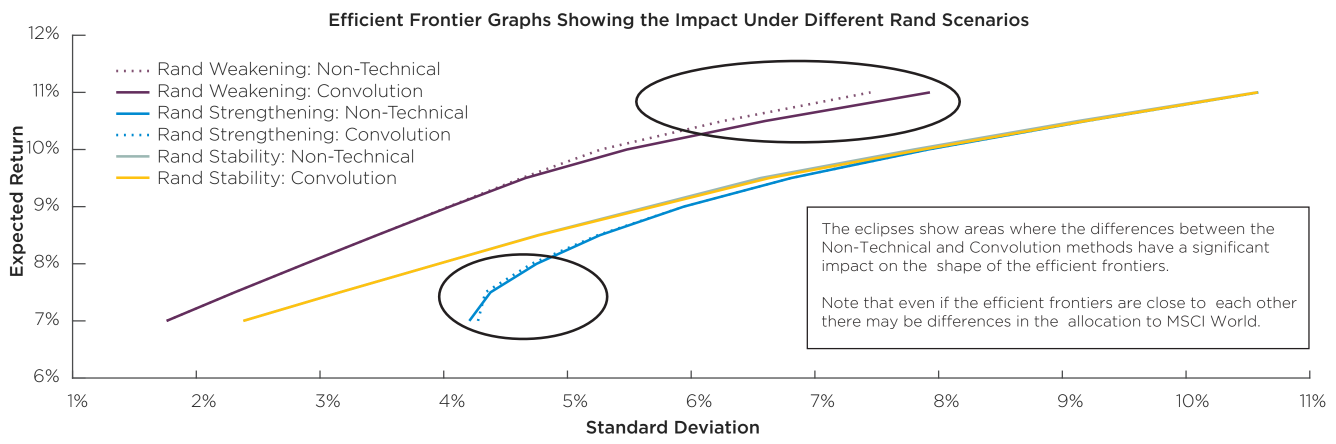
The third and final step results in an asset allocation, depending on the targeted average return of the investor. From the various estimated returns, we can determine how much we would allocate to the MSCI World under each of the Rand scenarios. For each scenario we indicate the resulting effects on portfolio allocation for the convolution method versus the traditional method, as in Table 8. The differences between each view are further highlighted along the efficient frontiers for each view in Figure 6.

TABLE 8:

Comparison of the percentage asset allocations awarded to the MSCI, depending on the Rand view, using the convolutions method and traditional method.

	Rand Weakening		Rand Stability		Rand Strengthening	
Return Target	Convolution	Non-technical	Convolution	Non-technical	Convolution	Non-Technical
8.00%	12.29%	10.80%	4.07%	5.16%	23.58%	20.48%
8.50%	16.33%	14.35%	5.85%	7.38%	14.43%	13.45%
9.00%	20.47%	17.99%	7.62%	9.61%	8.75%	7.98%
9.50%	24.61%	21.62%	9.40%	11.84%	3.07%	2.51%
10.00%	28.74%	25.26%	11.17%	14.06%	0.00%	0.00%
10.50%	30.00%	28.90%	10.41%	11.63%	0.00%	0.00%
11.00%	30.00%	30.00%	9.64%	9.19%	0.00%	0.00%
11.50%	30.00%	30.00%	8.03%	5.87%	0.00%	0.00%
12.00%	30.00%	30.00%	6.02%	2.98%	0.00%	0.00%

FIGURE 6: Efficient frontiers summarising the comparison between the different Rand view asset allocations via the traditional and convolution methods.



The impact on asset allocation

The differences made to asset allocations, based on future views of foreign asset returns, between the two methods ***are significant at points where the allocation differs by more than 1%***, shown on the efficient frontier.

In a Rand weakening scenario it becomes apparent that portfolio volatility is slightly reduced using the convolution method on the lower end of the efficient frontier, but this situation reverses towards the higher end of the efficient frontier as seen in Figure 6.

In a Rand stable environment it appears that the efficient frontiers are similar for both methods, except for the midsection of the frontier where a slight corner can be seen around the 10% expected return mark. It is clear that the allocation differences to the MSCI World would become significantly different here.

In a Rand strengthening environment, even with the volatility parameters being the same, we can see significant differences at the lower end of the efficient frontier regarding allocation for the Rand strengthening scenario. We believe that these differences are likely to disappear if we introduce other assets that have the same return profiles. Essentially, in certain situations, even if the convolution and the historic adjustment methods give similar parameters for the optimisation routine to use, the outcome could differ.

[9]For modelling the joint interaction between the foreign asset and the exchange rate, we provide further mathematical models in the appendix that assist with this.

Conclusion

It is apparent in the given examples that there are allocation differences that can play out when adjusting one's expectations. If expectations are adjusted slightly for each of the underlying assets there is a subtle difference in the final optimisation results. However, if the adjustments are significant then reliability is increased by using the convolution method. For instance, if we take the exchange rate's volatility per annum and adjust it slightly from 16.05% to 15.95% then the effect on asset allocation is minimal. However, if investors are interested in different asset with different dependencies, and they adjust the correlation coefficient from -0.4 to -0.7, and the volatility of the foreign asset from 10% to 12%, then again the convolution method offers an increase in reliability.

Our analysis using the convolution method allows for modelling each of the assets independently and then combining them in a useful manner. This can also then provide consistent views for an asset management firm where the team responsible for working on foreign exchange rate distributions (e.g. for pricing derivatives or trading) can share their model with the asset allocation team and thus a consistent house view can be obtained when making asset allocation decisions.

Closing Technical remark: **The convolution method is also able to give correct higher moments and can be used for more complicated modelling of skewness and kurtosis for each distribution**. It can therefore mix the distributions to obtain a sought-after distribution. This applies to other higher moments as well as lower/upper partial moments of all orders as well. Furthermore, in a multi-asset environment the convolution method can be extended to deal with the complexities of such an environment[9].

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